

# Modal Logic (80-315/80-615)

Spring 2022

Tuesday/Thursday 1:25–2:45, WEH 5320

<https://canvas.cmu.edu/courses/27646>

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**Course Description:** Standard logical languages can express negation ( $\neg p$ ), conjunction ( $p \wedge q$ ), material implication ( $p \rightarrow q$ ), quantification ( $\forall x p(x)$ ), etc. But they don't directly capture statements like the following: Alice knows  $p$ . Henceforth, it will be the case that  $p$ . It ought to be the case that  $p$ . If it had been the case that  $p$ , it would have been the case that  $q$ . Everybody knows  $p$ . Everybody knows that everybody knows  $p$ . Infinitely often in the future,  $p$  will be true. After an announcement of  $p$ , it will be the case that Alice knows  $q$ . If  $p$  is not permitted, then you ought to know that  $p$  is not permitted. etc.

Modal logic is a very general framework for systematically reasoning about statements like these. This course is an introduction to mathematical modal logic and its applications in philosophy, computer science, linguistics, and economics, with emphasis on epistemic interpretations (i.e., logics for representing and reasoning about knowledge/belief). We begin with a rigorous development of propositional modal logic: the basic language, interpretation in relational structures, axiom systems, proofs, and validity. We prove soundness and completeness of various systems using the canonical model method and study model equivalence and expressivity results. We also consider topological semantics as an alternative to relational semantics, and investigate the connection between the two.

In the latter part of the course we turn our attention to more specialized logical systems and their applications, as determined by the interests of the class. Topics may include: quantified modal logic, multi-agent systems and the notion of common knowledge (with applications to game theory), temporal and dynamic logics for (nondeterministic) program execution, logics for reasoning about counterfactuals, public announcement logic, deontic logic, intuitionistic logic, and others.

**Text:** Course notes will be provided; no textbook is required.

**Course Objectives:** The primary objective of this course is to develop *mathematical competence* in modal logic along with the ability apply this competence in useful ways. More precisely, this means being able to:

- navigate a variety of formal languages;
- interpret modalities in relational structures and explain the connection between formulas in the language and properties of the structures;
- formalize intuitions and analyze problems using modal logics;
- find axioms characterizing properties of important structures and prove completeness results.

**Evaluation:** Problem sets 60%; tests 30%; quizzes 10%.

Every other Thursday a problem set will be released, covering the previous 2 weeks of material, due 12 days after it is released (so always on a Tuesday).

- These are the foundation of the class—where I will push you to *really* engage with what you’ve learned. The bulk of the work for this course is here.
- You are encouraged to collaborate with classmates—just be sure to write up your own solutions, and clearly indicate on the first page with whom you have collaborated.
- All problem sets will be submitted via Gradescope, which is integrated with Canvas and can be accessed through the sidebar. They are due by the beginning of class; late submissions will be considered provided arrangements have been made in advance.

Each Tuesday there will be an in-class evaluation.

- On days when a problem set is due there will be a 20-minute test at the beginning of class. These tests are essentially in-class components of the problem sets; they cover exactly the same material but consist of shorter/easier problems.
- On the weeks in between problem set due dates, there will instead be a 10-minute quiz based on the material covered during the previous two weeks. Quizzes are not worth very much; they are intended primarily as self-assessment tools.

There is no final exam.

### **Engagement & Community:**

- To the extent you feel comfortable doing so, please try to be an active participant in class.
- Post on the Piazza forum freely and frequently—not only to ask questions but also to help answer questions that others have raised.
- Treat each other with respect. I take this very seriously and invite you to reach out to me if at any point you feel uncomfortable for any reason.

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For many of us, these are stressful times. My hope is for this class to be useful and interesting for you, and not yet another source of stress. If you find yourself struggling, you need help, or any aspect of this course is problematic for you (for any reason), please reach out to me. I’m also very happy to hear general feedback and constructive criticisms about the class at any time.

Take care of yourself. Try to get enough sleep, go for walks in the fresh air, and connect as you can with the people you love. Take time to relax. You can’t achieve your goals if you’re sick from stress or burnt out.

All of us benefit from support during tough times. You are not alone—an important part of the college experience is learning how to ask for help. If you or anyone you know needs help, consider reaching out to a friend, faculty member, or family member you trust. Counseling and Psychological Services (CaPS) is also here to help: call 412-268-2922 or visit their website at <https://www.cmu.edu/counseling/>.

Finally, if you are worried about affording food or feeling insecure about food, there are resources on campus who can help. Email the CMU Food Pantry Coordinator to schedule an appointment:

Pantry Coordinator  
cmu-pantry@andrew.cmu.edu  
412-268-8704 (SLICE office)

**Course Outline:** Weekly topics are subject to change; the below should only be considered a rough guideline. The evaluation schedule is fixed.

<b>Date</b>	<b>Topic</b>	<b>Evaluation</b>
1/18 1/20	introduction and motivation · relational structures	– –
1/25 1/27	modal languages · models and frames · validity	quiz 1 PS1 assigned
2/1 2/3	frame definability	quiz 2 –
2/8 2/10	invariance · bisimulation · bounded morphisms	PS1 due; test 1 PS2 assigned
2/15 2/17	frame undefinability · expressivity	quiz 3 –
2/22 2/24	axioms and proofs · soundness	PS2 due; test 2 PS3 assigned
3/1 3/3	the canonical model · completeness	quiz 4 –
3/8 3/10	<i>no class (spring break)</i>	– –
3/15 3/17	filtration and decidability	PS3 due; test 3 PS4 assigned
3/22 3/24	topology	quiz 5 –
3/29 3/31	topological semantics	PS4 due; test 4 PS5 assigned
4/5 4/7	epistemology through topology <i>no class (spring carnival)</i>	quiz 6 –
4/12 4/14	topics (e.g., multi-agent epistemic logic, common knowledge)	PS5 due; test 5 PS6 assigned
4/19 4/21	topics (e.g., public announcements, doxastic logics)	quiz 7 –
4/26 4/28	topics (e.g., intuitionistic logic)	PS6 due; test 6 –

## Campus Resources

**Academic Integrity** Carnegie Mellon's Policy on Academic Integrity is available here: <http://www.cmu.edu/policies/student-and-student-life/academic-integrity.html>. It applies to all courses at this university, including this one.

**Student Academic Success Center (SASC):** SASC focuses on creating spaces for students to engage in their coursework and approach learning through a variety of group and individual tutoring options. They offer many opportunities for students to deepen their understanding of who they are as learners, communicators, and scholars. Their workshops are free to the CMU community and meet the needs of all disciplines and levels of study. SASC programs to support student learning include:

- Academic Coaching  
<https://www.cmu.edu/student-success/programs/coaching.html>
- Peer Tutoring  
<https://www.cmu.edu/student-success/programs/tutoring.html>
- Communication Support  
<https://www.cmu.edu/student-success/programs/communication-support/index.html>
- Language and Cross-cultural Support  
<https://www.cmu.edu/student-success/programs/language-support/index.html>
- Supplemental Instruction  
<https://www.cmu.edu/student-success/programs/supp-inst.html>

For more information, visit <https://www.cmu.edu/student-success/>.

**Disability Services:** The Office of Disability Resources at Carnegie Mellon University has a continued mission to provide physical and programmatic campus access to all events and information within the Carnegie Mellon community. They work to ensure that qualified individuals receive reasonable accommodations as guaranteed by the Americans With Disabilities Act (ADA) and Section 504 of the Rehabilitation Act of 1973. For more information, visit: <http://www.cmu.edu/disability-resources/>.

If you have a disability and have an accommodations letter from the Disability Resources office, I encourage you to discuss your accommodations and needs with me as early in the semester as possible. I will work with you to ensure that accommodations are provided as appropriate. If you suspect that you may have a disability and would benefit from accommodations but are not yet registered with the Office of Disability Resources, I encourage you to contact them at [access@andrew.cmu.edu](mailto:access@andrew.cmu.edu).